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# CONTACT-IMPACT ALGORITHMS FOR PENETRATION STUDIES

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19. ABSTRACT (Continue on reverse if necessary and identify by block number)  Contact impact algorithms, which are sometimes called slideline algorithms, are a computationally time-consuming part of many explicit simulations of nonlinear problems. One of the reasons for their computational expense is that slideline algorithms are complex with many branches, so they are not amenable to vectorization, which is essential for speed on supercomputers. The pinball algorithm is a simplified slideline algorithm which is readily vectorized. Its major idea is to embed pinballs in surface elements and to enforce the impenetrability condition only to pinballs. The algorithm has been implemented in the WHAMS-3D code. Examples of solutions and running times are given.				
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## PREFACE

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## 1. Introduction

In this paper, a new contact-impact procedure called the pinball algorithm is described, for previous studies of contact-impact see [1-4]. A short description of pinball was previously given by Belytschko and Neal[5]. The thrust of the pinball algorithm is to allow vectorization of as much of the sideline calculations as possible. This is accomplished by greatly simplifying both the search for the elements involved in the impact and in the enforcement of impenetrability with the use of spheres, or pinballs, for each element in the sideline calculations. In this way, the search requires a simple check on the distances between pinballs to determine interpenetration. Once the contacting pairs of pinballs have been determined, the impenetrability condition is enforced with the use of a penalty formulation which can be completely vectorized. A similar idea has also been used in the two-dimensional NABOR algorithm[6], but the NABOR method used an ad hoc method based on spheres for the determination of stresses in the continua and did not use a representation of the surface normal. In the pinball algorithm the element spheres are used only in the contact algorithm, while standard continuum mechanics is used for the continuum elements.

## 2. Variational Inequality and Discrete Interpolants

The weak form of the contact problem is obtained from the principle of virtual work by appending the Lagrange multiplier type term  $\delta(\lambda \dot{g})$ . We consider the trial functions to be kinematically admissible functions  $v$ , so  $v_i \in V$  and  $\lambda \in \Lambda$  where

$$V = \{v_i : v_i \in C^0(\Omega^A \cup \Omega^B), v_i = v_i^* \text{ on } \Gamma_{v_i}\} \quad (2.1a)$$

$$\Lambda = \{\lambda : \lambda \in C^{-1}(\Gamma_c), \lambda \leq 0\} \quad (2.1b)$$

As indicated above, these functions need only be piecewise continuous and satisfy essential boundary conditions. The variations (or test functions)  $\delta v_i \in V_0$ ,  $\delta \lambda \in \Lambda_0$  where

$$V_0 = \{\delta v_i : \delta v_i \in C^0, \delta v_i = 0 \text{ on } \Gamma_{v_i}\} \quad (2.2a)$$

$$\Lambda_0 = \{\delta \lambda : \delta \lambda \in C^{-1}, \delta \lambda \leq 0 \text{ on } \Gamma_c\} \quad (2.2b)$$

$$\delta W = \delta W^{\text{int}} + \delta M - \delta W^{\text{ext}} \quad (2.3)$$

(see Belytschko [7]) where

$$\delta W^{\text{int}} = \int_{\Omega} \delta v_{(i,j)} \sigma_{ij} d\Omega \quad (2.4)$$

$$\delta W^{\text{ext}} = \int_{\Omega} \delta v_i b_i d\Omega + \int_{\Gamma_{\tau_j}} \delta v_j \tau_j^* d\Gamma \quad (2.5)$$

$$\delta M = \int_{\Omega} \delta v_i \rho \dot{v}_i d\Omega \quad (2.6)$$

where the density is denoted by  $\rho$ , the body force by  $b_i$ , the surface tractions by  $\tau_i^*$ , and the Lagrange multipliers by  $\lambda$ . The stress state is described by the Cauchy (physical) stress  $\sigma_{ij}$ .

The weak form for the contact problem is then given by :

If  $v \in V$ ,  $\lambda \in \Lambda$  and

$$\delta W + \int_{\Gamma_c} \delta(\lambda \bar{g}) d\Gamma \geq 0 \quad (2.7)$$

for all  $\delta v \in V_0$ ,  $\delta \lambda \in \Lambda_0$ , then the momentum equation, traction boundary conditions and contact surface inequalities are satisfied. The equivalence of this weak form to the governing partial differential equations is demonstrated in [5].

### 3. Pinball Algorithm

The main idea of the pinball algorithm is to enforce the impenetrability condition and contact conditions on a set of spheres, or pinballs which are embedded in the finite elements. By enforcing the contact constraint on the spheres rather than the elements themselves, the time required by the contact algorithm can be greatly reduced because: (1) the determination of whether interpenetration has occurred becomes a simple check of the distance between two pinballs, (2) when combined with a penalty method, it involves almost no recursive calculations or conditional statements, so it is much more amenable to vectorization.

The hexahedral elements used in this formulation are described in detail by Flanagan and Belytschko[13]. A sphere, or pinball, is embedded in each of these hexahedral elements of the mesh. These pinballs will then be used to determine which elements are involved in the contact. The center and radii of the sphere are given in element  $e$  by

$$C_i = \frac{1}{8} \sum_{I=1}^8 x_{Ii}^e \quad (3.1)$$

$$R = \sqrt[3]{\frac{3V^e}{4\pi}} \quad (3.2)$$

respectively, where  $C_i$  are the coordinates of the center of the sphere,  $x_{Ii}^e$  are the coordinates of node  $I$  of element  $e$ ,  $R$  is radius of the pinball, and  $V^e$  is the volume of element  $e$ .

The center of each sphere is simply the average of its nodal coordinates while the radius is determined by setting the volume of the resulting sphere equal to the volume of the element itself. For materials with substantial compressibility the radius for each element would have to be recalculated every few steps.

The detection of the impacting pairs is, computationally, a very simple procedure. The distance between the centers of each slave pinball and each master pinball is calculated and then compared with the sum of the radii of the two elements. Interpenetration has occurred when

$$d_{ij} < R_i + R_j \quad (3.3)$$

where  $d_{ij}$  is the distance between the centers of elements 1 and 2 and  $R_i, R_j$  are the radii of elements  $i$  and  $j$ . Note that in this procedure the masters and the slaves may be penetrated by more than one element during a time step.

In a penalty of velocity projection formulation, where forces are added to each node involved in contact, contact of one element with several other elements does not impair the algorithm. On the other hand, in the displacement projection form of the Lagrange multiplier method[4], since the nodes are moved, difficulties arise when a single element comes into contact with several other elements because nodes are displaced in the contact algorithm. This difficulty is illustrated in figures 1. In figure 1a element 1 contacts two other elements 2 and 3. If a displacement-based slideline algorithm is used, element 1 would first be moved so that it does not penetrate element 2 as in figure 1b, and then it would be moved again as shown in figure 1c so as not to penetrate element 3. During this second adjustment element 1 may again penetrate element 2 and this penetration would go undetected. This problem is especially troublesome in vectorized algorithms. Since the adjustment of position is a recursive operation, the first adjustment would not take place at all.

The penalty force is proportional to the depth of penetration between the two elements; therefore the next step of the procedure is to determine the penetration depth of the two elements. In the pinball algorithm, the penetration is easily calculated. Consider two interpenetrating pinballs, 1 and 2, in figure 2, with the velocities  $v_1$  and  $v_2$ ; the normal of the associated surfaces are  $n_1$  and  $n_2$ . The position vectors of the centers of the two pinballs are given by  $C_1$  and  $C_2$ . The penetration is given by  $g$  and is defined as the relative displacement of the centers of the pinballs in the average normal direction needed to eliminate interpenetration, so that the following equation determines  $g$

$$\|C_1 - C_2 + gn\|^2 = (R_1 + R_2)^2 \quad (3.4)$$

and the normals of the two elements are given by

$$n = \frac{1}{2}(n_2 - n_1) \quad (3.5)$$

This gives

$$g = -b + \sqrt{b^2 - c} \quad (3.6)$$

where

$$b = \frac{n_i(x_i - C_{2i})}{n_j n_j}$$

$$c = \frac{x_i x_i + C_{2i} C_{2i} - 2x_i C_{2i} - (R_1 + R_2)^2}{n_j n_j} \quad (3.7)$$

Note that only the positive sign on the radical in (6) need be considered; the negative root corresponds to a negative value of  $g$  which is irrelevant.

The penalty force which will be applied to the nodes of each element is proportional to the penetration depth and is given by

$$F^P = (p_1 \dot{g} + p_2 g) n \quad (3.8)$$

An expression for the parameter,  $p_2$ , has been proposed for the case of a node impacting the surface of an element and the volume, area and bulk modulus referred to the impacted element[16]

$$p_2 = \frac{\beta B A^2}{V} \quad (3.9)$$

where  $B$ ,  $A$ , and  $V$  are the bulk modulus, area of the impacted surface, and volume of the element,  $F^P$  is the penalty force on the pinball.

The rate of penetration is computed by

$$\dot{g} = \frac{1}{8} \sum_{I=1}^8 (v_I^2 - v_I^1) \cdot n \quad (3.10)$$

In the present context, the properties of two pinballs must be considered, so the penalty parameter will be given by

$$p_2 = \frac{1}{2} \beta (B_1 R_1 + B_2 R_2) \quad (3.11)$$

where  $B_1$ ,  $B_2$  are the bulk moduli of the impacting pinballs, and  $R_1$ ,  $R_2$  are the radii of the impacting pinballs. Equation (9) gives the contact force that will be applied in opposite directions to each of the two impacting pinballs. This force is then divided among the eight nodes of each element.

$$F_n^{eP} = \frac{1}{8} F^P \quad n = 1,8 \quad (3.12)$$

where  $F_n^{eP}$  are the element level penalty force on local node  $n$  of the element. These forces are then assembled to the global force vector as usual. A flowchart of the impact algorithm is given in figure 3.

The penalty force is divided among the eight nodes of the hexahedron to preserve the symmetry of the underlying linearized system. Since the position of the pinball depends on the eight nodes of the hexahedron, the linearized equations would not be symmetric if the force were subdivided only among the surface nodes; an alternative algorithm where  $C$  depends only on the surface node velocities and hence the penalty forces are distributed only to the surface nodes is now under investigation, Belytschko and Bindeman[17].

The penalty forces, along with the forces arising from element stresses and externally applied loads, are used in the calculation of the nodal accelerations. Therefore the contact routine appears in the algorithm immediately before the nodal accelerations are calculated. The flowchart of the complete explicit algorithm with the contact algorithm are given in figure 4.

#### 4. Numerical Examples

In order to test the accuracy and efficiency of the proposed contact procedure several example problems were examined. The first problem considered was the impact of two one-dimensional bars consisting of ten elements each. This problem was performed in order to compare two different methods of enforcing the impenetrability constraint: the penalty method and the projection method. This contact constraint is the only nonlinearity that appears in the problem.

As can be seen in figure 5 one of the bars is given an initial velocity so that it impacts with the second bar. The material properties are such that the wave speed in the two bars is 10.0 m/s.

Figure 6 through 8 give the velocity time histories for the nodes at the midpoint of the first rod ( $x=5.0$ ), at the interface on the first rod ( $x=10.0$ ), and at the midpoint of the second rod ( $x=15.5$ ). As can be seen from figure 7, the penalty method gives a rather noisy solution at the contact interface yet this does not appear to have much affect away from the contact zone (see figures 6 and 8). The results for the Lagrange multiplier method were nearly identical to the projection method and therefore were not included in the results.

The second problem considered was of a copper rod impacting a steel plate at high velocity. The rod, acting as the projectile, consisted of 414 elements while the plate or target was made up of 1,428 elements. The geometry and the material properties for each of the objects is given in table 1. The evolution of the problem is shown in figure 9. In this example problem and the one that follows a Von Mises yield criteria is used with a piece-wise linear stress-strain model. Once the effective stress reaches yield the material undergoes isotropic hardening until the effective stress is equal to the ultimate stress, at which point the material is considered perfectly plastic. When the effective strain of an element reaches the maximum allowable effective strain, the element is eroded, that is the stress in the element is considered zero from that time on. The maximum allowable effective strain used for steel is 1.0 while that used for copper is 2.0.

This problem was also examined by Belytschko and Lin[4] with their projection method and a comparison of running times for both the methods is given in the first column of table 2. As can be seen from this table, the efficiency of the new algorithm is substantially better than the previous one on a vectorized machine. For this comparison, both algorithms were implemented into the three dimensional finite element code WHAM3D and run on a Cray X-MP/14 with the CFT77 compiler. The differences shown in running times is due only to the different slideline algorithms used.

The importance of vectorization for both of these slideline algorithms is demonstrated graphically in figures 10 and 11. In these figures a breakdown of the total CPU is given for each part of the program for both vectorized and unvectorized compilers. (The unvectorized runs were performed on the same machine using the CFT77 compiler with the vectorization turned off.) For unvectorized runs, the new algorithm is only marginally more efficient than the previous method. When the vectorized compiler is used, however, the old version of the slideline algorithm consumes nearly fifty percent of the total CPU, the new procedure this value has been reduced to only fifteen percent for the vectorized run.

The impact of an elastic sphere with a rigid wall was examined to test the accuracy of the new algorithm. Ten degrees of the sphere are modelled with 499 elements as shown in figure 12. All of the nodes in this model are constrained in the circumferential direction; in addition, the nodes along the diameter are also constrained in the radial direction. Figure 12 also gives the material properties and dimensions of the sphere. The contact radius as a function of time is compared for the numerical simulation and the analytical result[21] in figure 13.

The final example problem is that a rod striking a plate at 136,000 cm/sec and a  $60^\circ$  obliquity. The projectile is modeled with 414 elements and the target with 5,300 elements. The time duration of the simulation is  $1.0 \times 10^{-4}$  sec. The geometry and material properties are described in table 3. The initial mesh is shown in figure 14. The final mass of the projectile is 42% of the initial mass and the exit velocity is 117,756.34 cm/sec in the x direction.

## 5. Conclusions

The major breakthrough of this paper is the demonstration that a contact-impact algorithm can be simplified dramatically by interpreting the gap  $g$  between the bodies as the gap between spheres embedded in the elements. This simplifies the contact-impact algorithm and facilitates vectorization. Computer times for large three-dimensional problems show a fivefold speedup in the slideline algorithm and as much as a factor of two in total running time

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Table 1. Geometry and material properties of penetration problem 1.

	<u>Projectile(rod with a round nose)</u>	<u>Target(plate)</u>
Length	= 4.900 in.	3.950 in.
Width	= -	7.900 in.(half plate is modeled)
Thickness	= -	0.375 in.
Radius	= 0.500 in.	-
Density	= $8.31\text{e-}3 \text{ lb-sec}^2 / \text{in.}^4$	$7.34\text{e-}3 \text{ lb-sec}^2 / \text{in.}^4$
Bulk Modulus	= $2.0739\text{e+}7 \text{ psi}$	$2.4200\text{e+}7 \text{ psi}$
Shear Modulus	= $6.3800\text{e+}6 \text{ psi}$	$9.3000\text{e+}6 \text{ psi}$
Plastic Modulus	= $1.5000\text{e+}5 \text{ psi}$	$1.4300\text{e+}5 \text{ psi}$
Yield Stress	= $2.0300\text{e+}4 \text{ psi}$	$1.6000\text{e+}5 \text{ psi}$
Ultimate Stress	= $6.5300\text{e+}4 \text{ psi}$	$2.0300\text{e+}5 \text{ psi}$
Initial Velocity	= $5.5566\text{e+}4 \text{ in./ sec. (x-component)}$ $-5.5566\text{e+}4 \text{ in./ sec (z-component)}$	0.0

Table 2. Timing studies for penetration problems.

<u>Algorithm</u>	<u>Example 1</u>	<u>Example 2, mesh 1</u>	<u>Example 2, mesh 2</u>
Previous method	34.7 sec.	94.4 sec.	302.2 sec.
Pinball algorithm	22.0 sec.	10.4 sec.	143.0 sec.

Table 3. Geometry and material properties of penetration problem 4

	<u>Projectile(rod with a round nose)</u>	<u>Target(plate)</u>
Length	= 10.25 cm	20.00cm
Width	= -	10.00cm
Thickness	= -	2.53cm
Radius	= 0.51cm	-
Density	= $7.77\text{e-}3 \text{ kgm/cm}^3$	$7.77\text{e-}3 \text{ kgm/cm}^3$
Young's Modulus	= $2.07\text{e+}7 \text{ N/cm}^2$	$2.07\text{e+}7 \text{ N/cm}^2$
Yield Stress	= $1.38\text{e+}5 \text{ N/cm}^2$	$9.13\text{e+}4 \text{ N/cm}^2$
Ultimate stress	= -	$1.12\text{e+}5 \text{ N/cm}^2$
Failure stress	= -	$3.65\text{e+}4 \text{ N/cm}^2$
Failure Strain	= 2.0	2.5
Initial Velocity	= $117756.34 \text{ cm/sec. ( x-component)}$ $-68015.79\text{cm/sec ( y-component)}$	

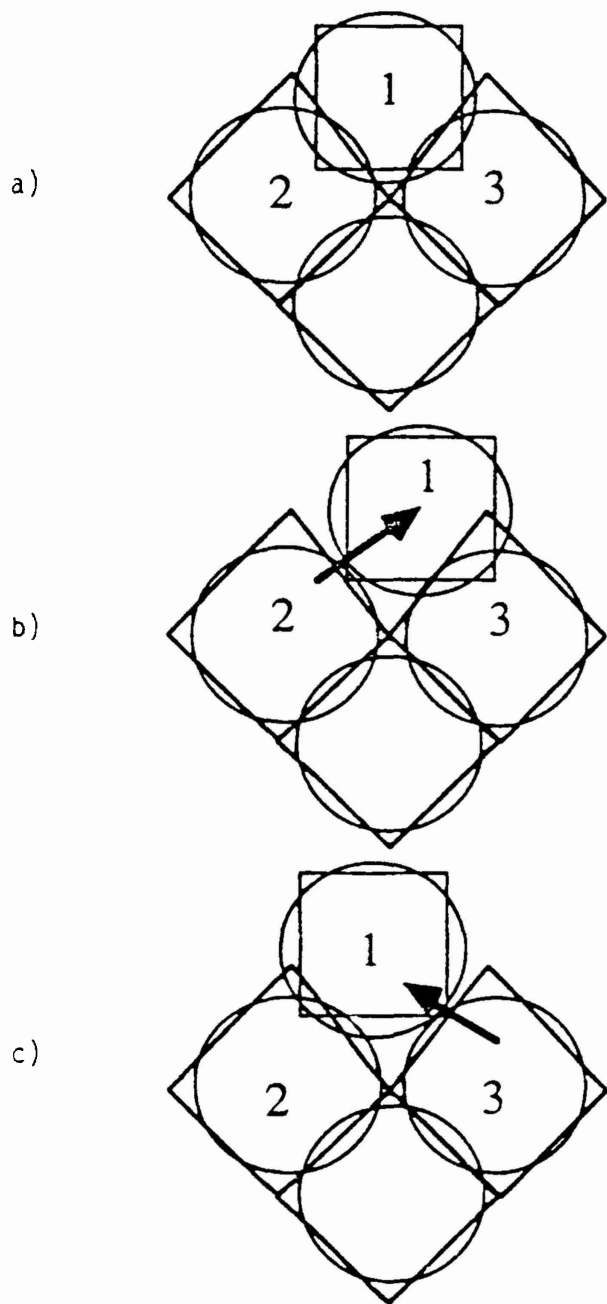


Fig. 1. a) Impact of one element with two other elements;  
 b) movement of element 1 due to impact with element 2;  
 c) movement of element 1 due to impact with element 3.

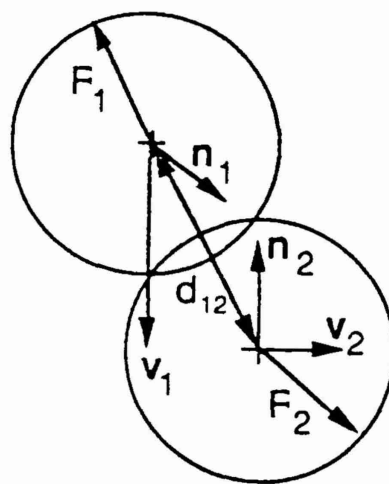


Fig. 2. Interpenetration of two pinballs.

1. If this is the first step, use the element volume to calculate a radius for all elements on the slideline.
2. Calculate element normals for all elements. Elements with zero normals are eliminated from consideration in the contact search.
3. Calculate the center of all elements with non-zero normals.
4. Put elements into appropriate cells.
5. Loop through elements of each cell to determine the penetrating pairs of elements.
6. Calculate the contact forces to be applied to the nodes of impacting element pairs.
7. Return to main driving routine.

Fig. 3. Pinball algorithm.

1. Initialization.
2. Calculate the external nodal forces.
3. Compute the internal nodal force array.
  - a. Calculate the element stresses.
  - b. Compute the element nodal forces arising from the element stresses.
  - c. Assemble the element nodal forces to the internal nodal force array.
4. Call the slideline algorithm to calculate the contact forces and add them to the external force array.
5. Compute the nodal accelerations.
6. Integrate the accelerations to obtain the nodal velocities and displacements.
7. Go to 2.

Fig. 4. Explicit time stepping procedure including slideline procedure.

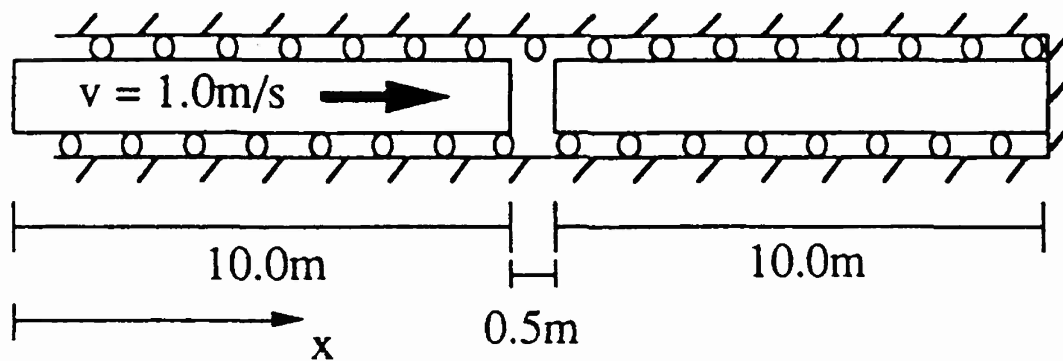


Fig. 5. One-dimensional impact problem.

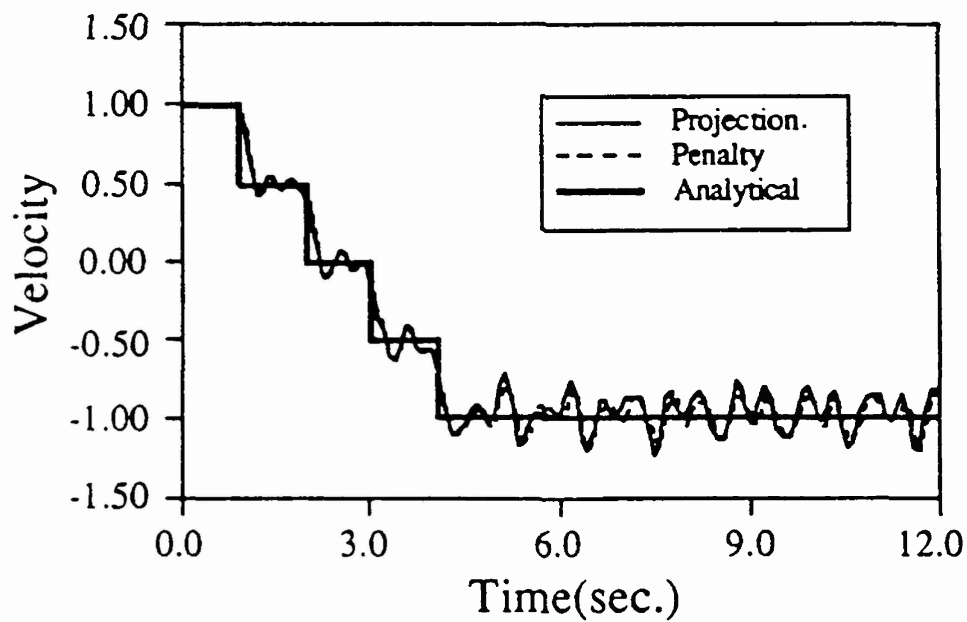


Fig. 6. Projection and penalty methods at midpoint of first rod.

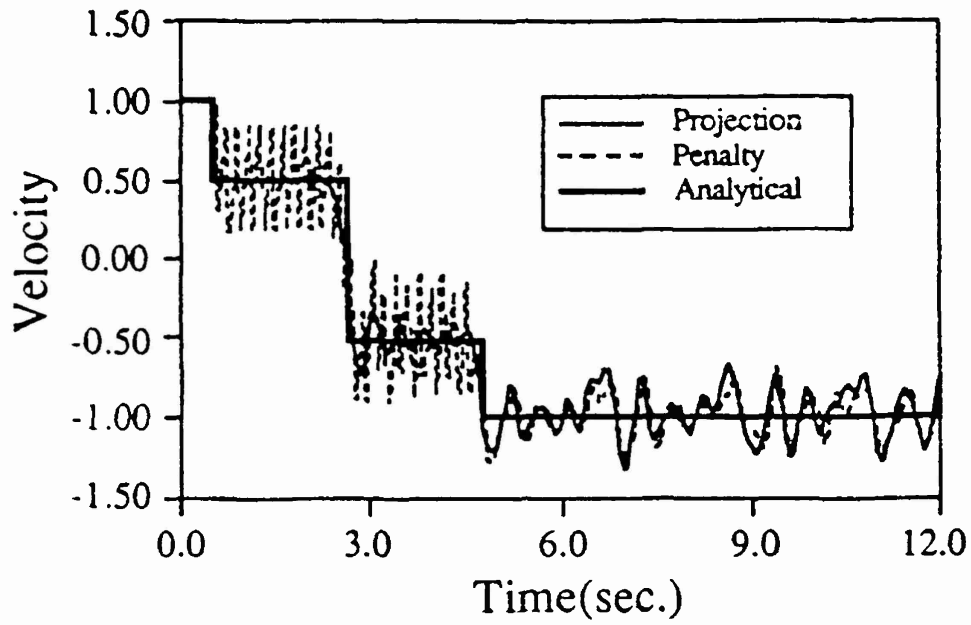


Fig. 7. Projection and penalty methods at interface on first rod.

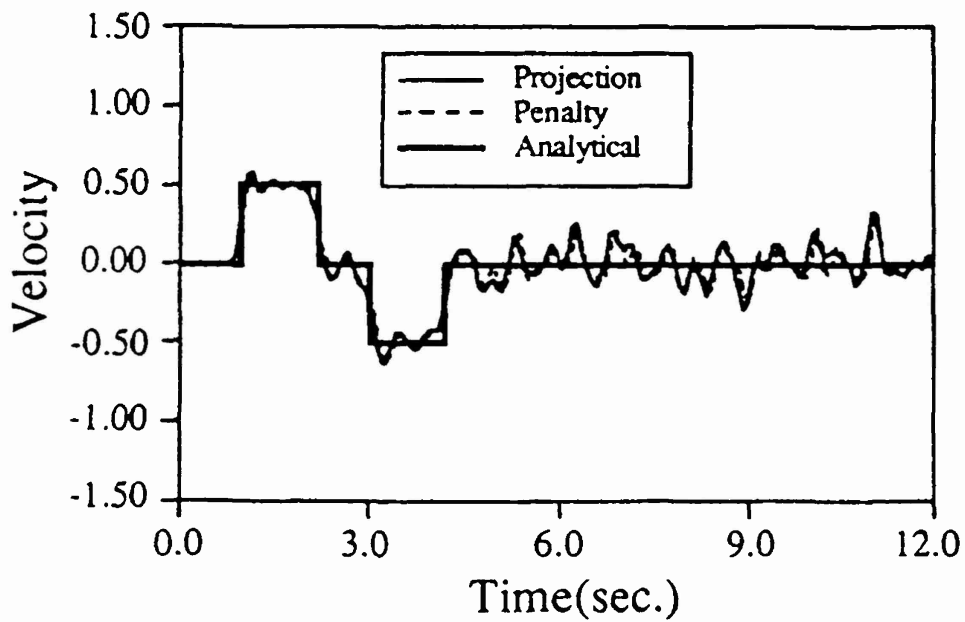


Fig. 8. Projection and penalty methods at midpoint of second rod.

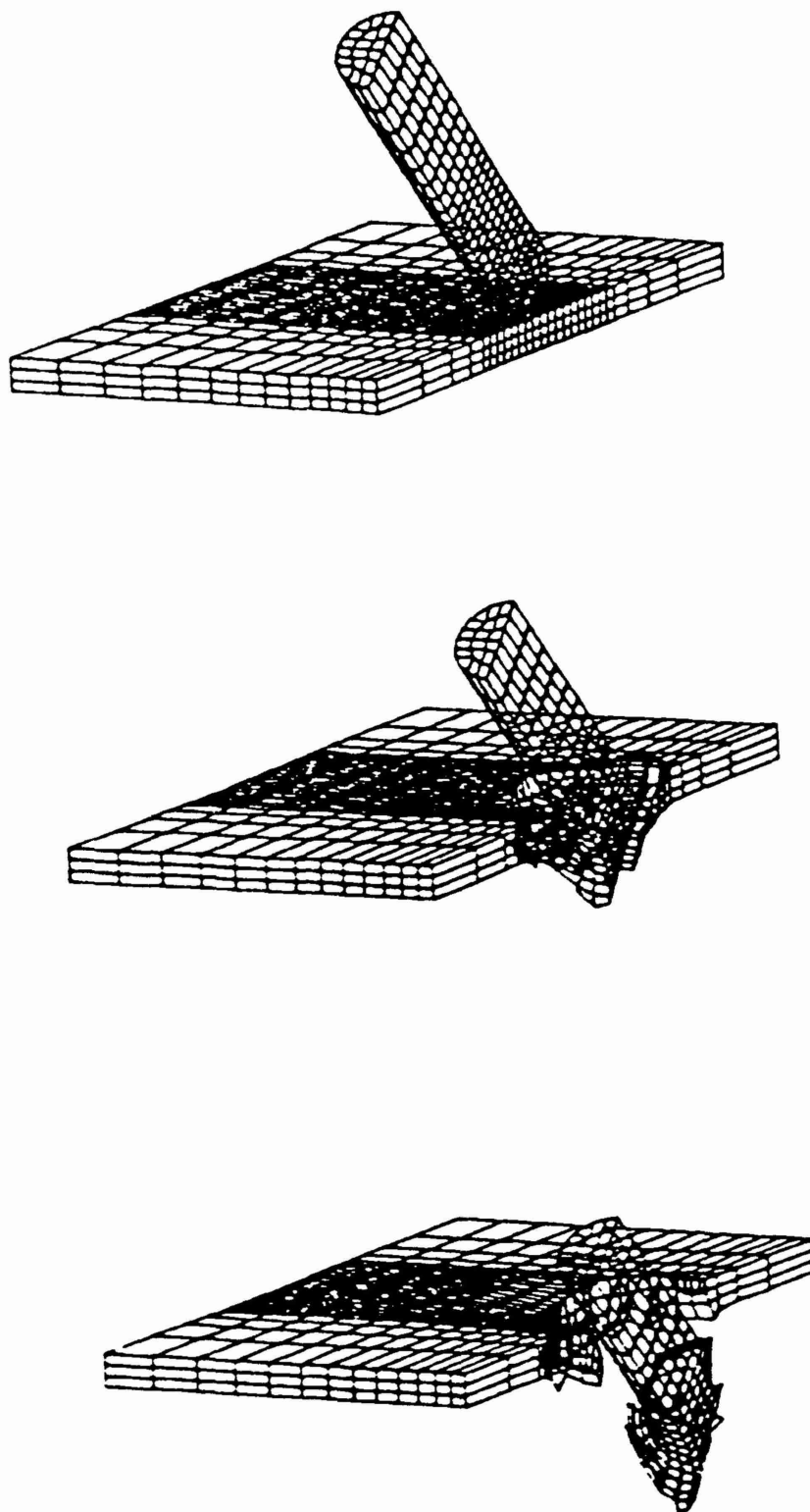


Fig. 9. Example problem 1 at times 0, 25, and 54  $\mu$ seconds.

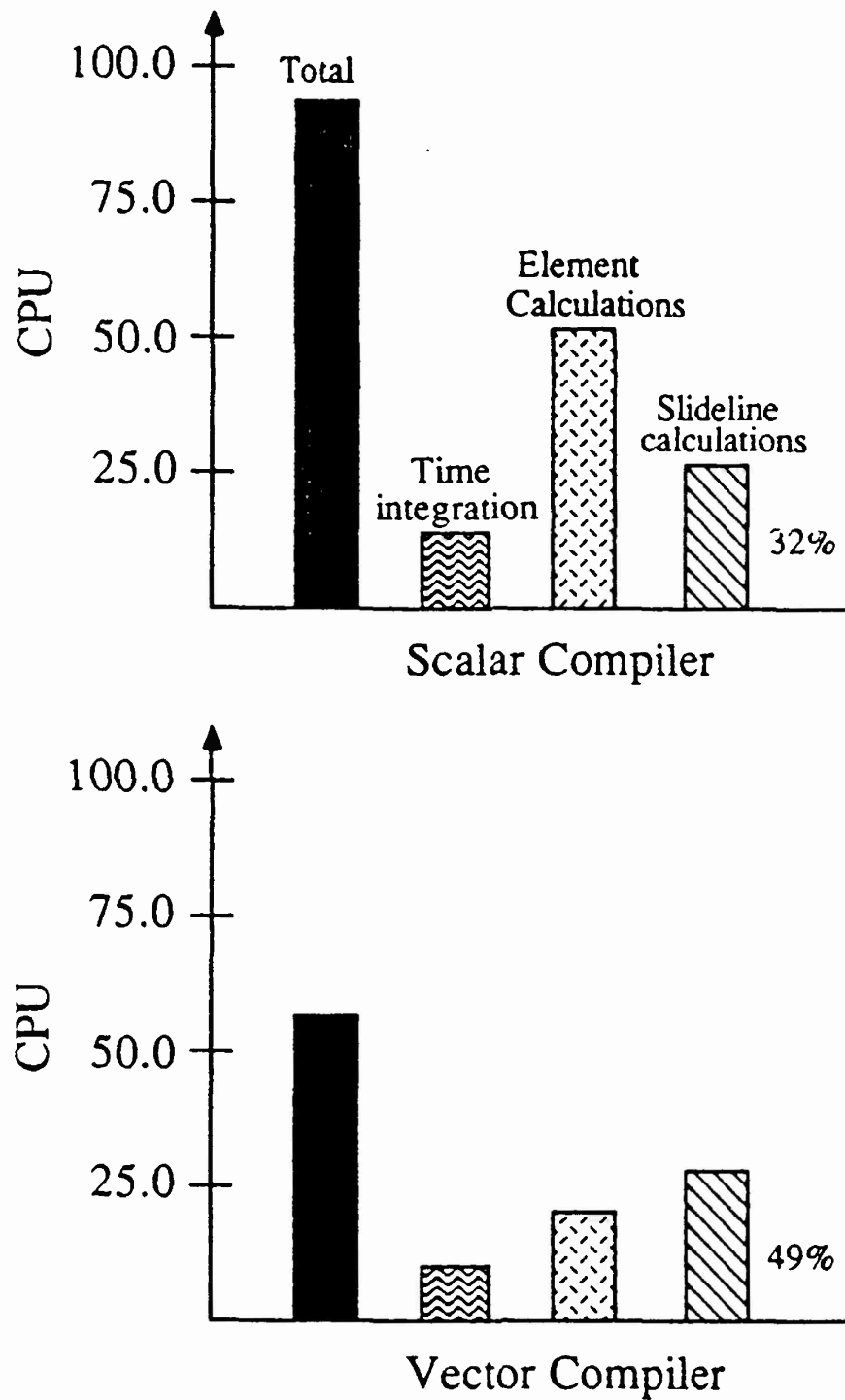


Fig. 10. CPU requirements of the Belytschko-Lin algorithm.

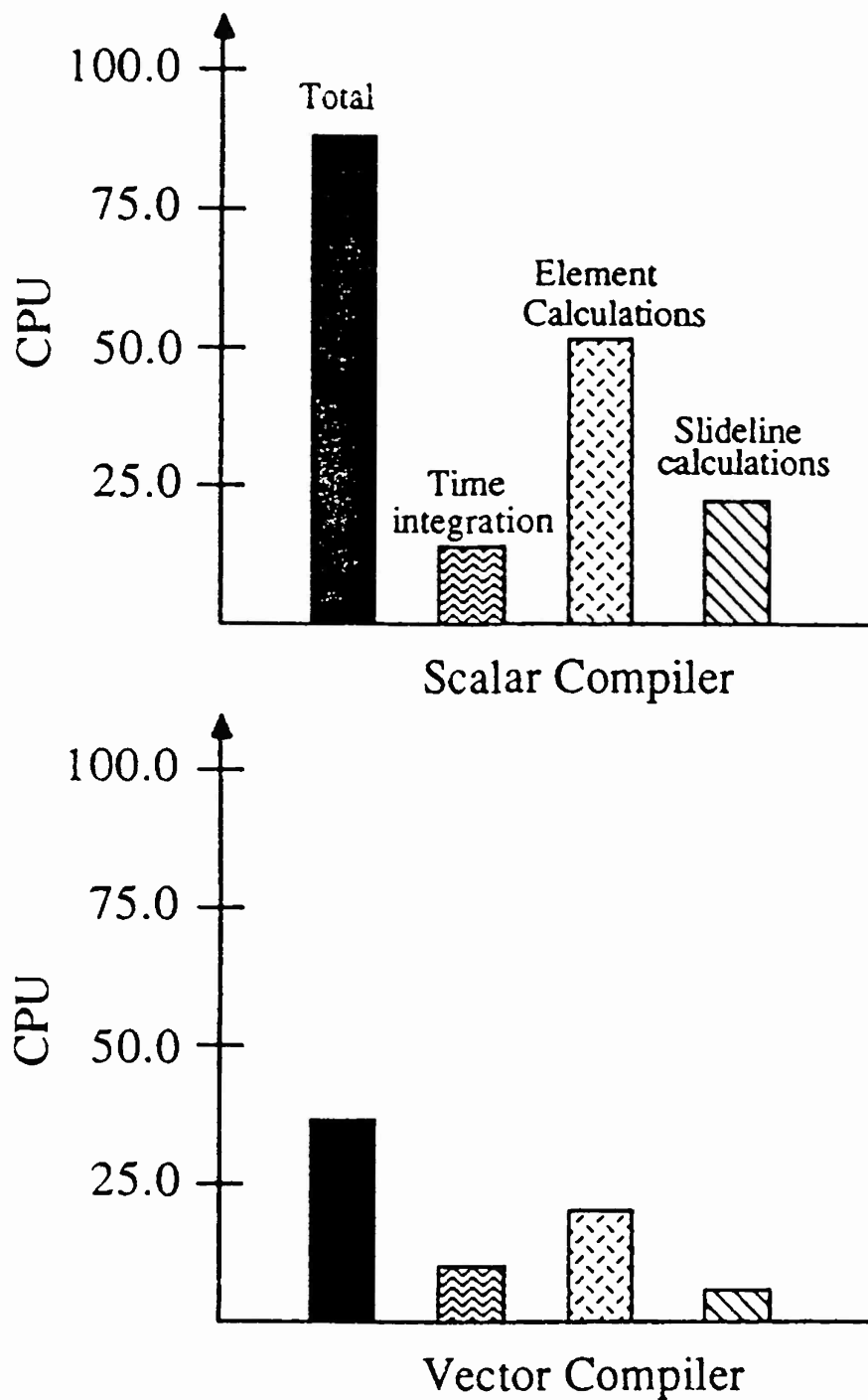
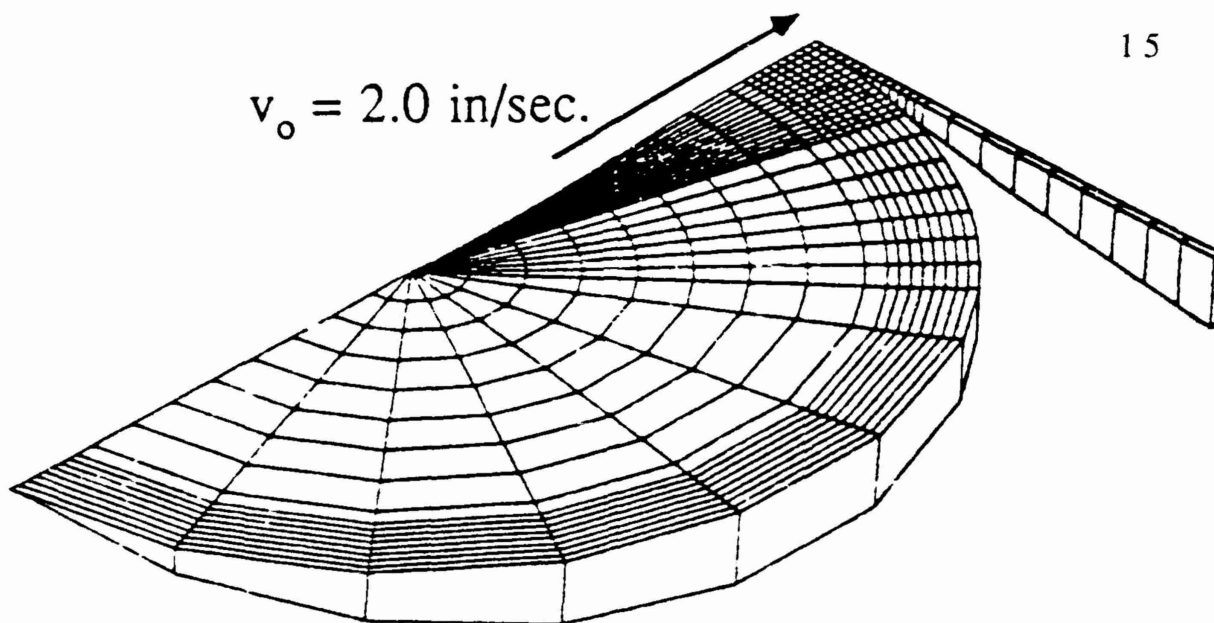


Fig. 11. CPU requirements of the pinball algorithm.



$$E = 1000 \text{ psi}$$

$$\rho = 0.01 \text{ lbf-sec}^2/\text{in}^4$$

$$\nu = 0.3$$

$$v_o = 2.0 \text{ in/sec.}$$

$$\text{Radius} = 5.0 \text{ in}$$

Fig. 12. Elastic impact of sphere with rigid wall.

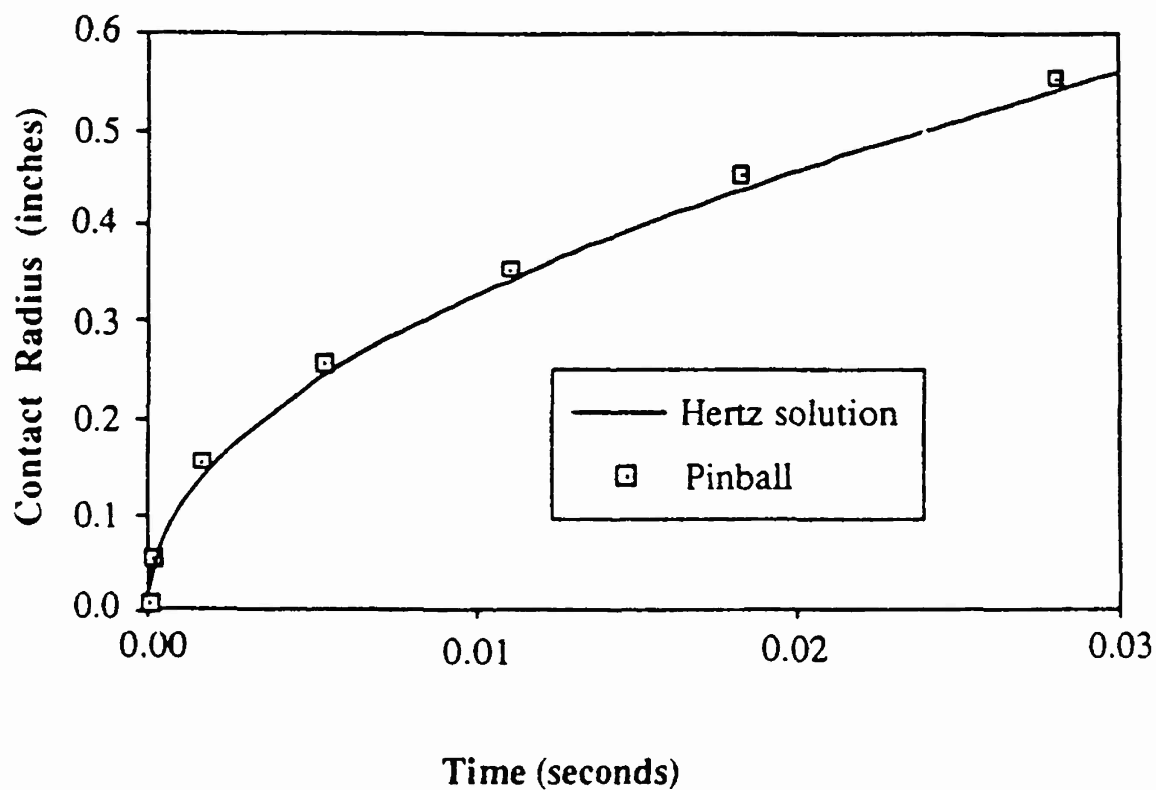


Fig. 13. Radius of contact.

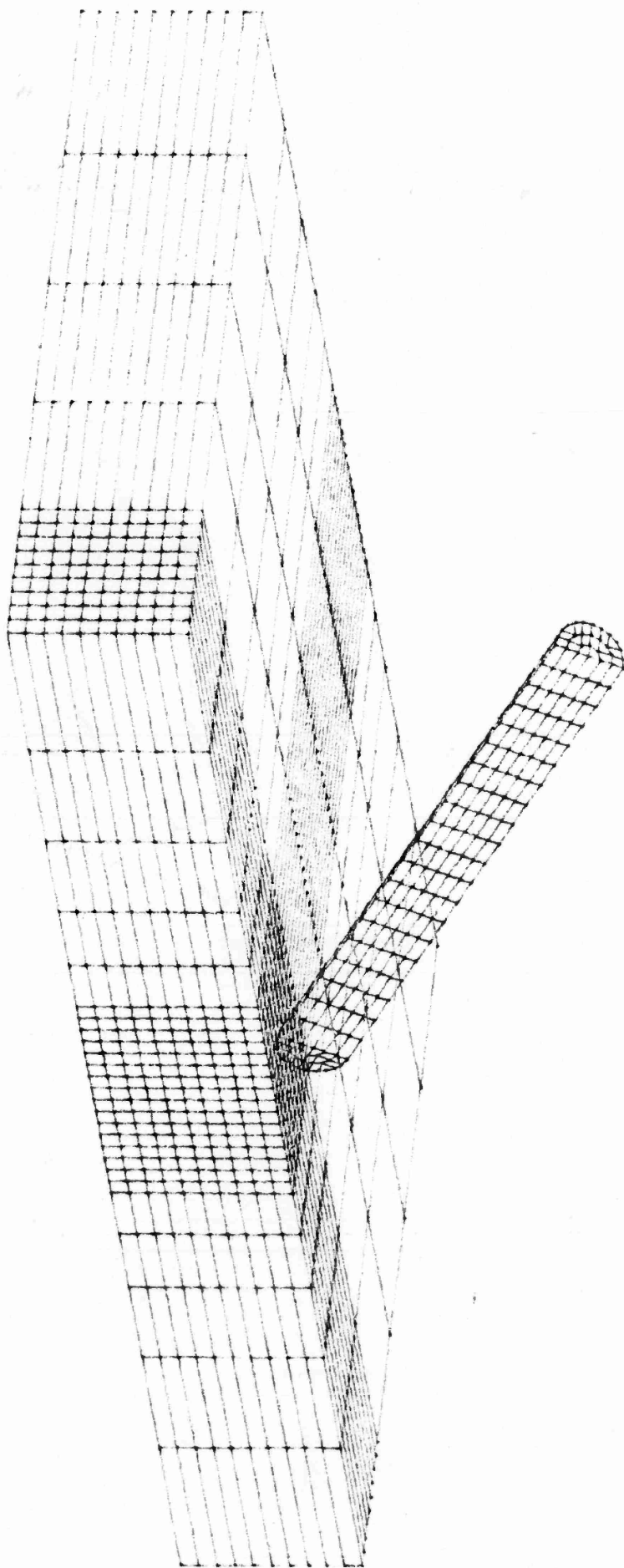


Fig. 14. The initial mesh of a rod striking a plate.